Phonon-Induced Resistivity of Electron Liquids in Quantum Wires

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We study the resistivity of a quantum wire caused by backscattering of electrons by acoustic phonons. In the presence of Coulomb interactions, backscattering is strongly enhanced at low temperatures due to Luttinger liquid effects. Information about the strength of the interactions can be obtained from a measurement of the temperature dependence of the resistivity.

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Interacting electrons in one dimension form a Luttinger liquid with properties very different from those of the more familiar Fermi liquid [1]. For one, the low-energy excitations of a Luttinger liquid are not fermionic quasiparticles, but rather collective spin- and charge-density modes. In addition, correlation functions, such as the density-density correlator, decay as power laws with nonuniversal exponents depending on the strength of the interactions. Ballistic quantum wires [2,3] provide a highly controllable experimental realization of a one-dimensional (1D) electronic conductor, and very low electron densities favorable for the observation of interaction effects can be routinely achieved. Experiments on quantum wires thus allow us to test the predictions of the Luttinger liquid theory, and indeed observations of the power-law decay of correlation functions [4] and of spin-charge separation [5] have been reported.

An interesting and well-known consequence of electron-electron interactions in one dimension is the enhanced electron backscattering by an impurity [6] in a Luttinger liquid. In this Letter we show that backscattering of the electrons in a quantum wire by the three-dimensional phonons of the surrounding semiconductor heterostructure is also strongly enhanced by interactions. A measurement of the phonon-induced resistivity provides information about the strength of electron-electron interactions and thus allows us to further test Luttinger liquid theory. Prior investigations of the interplay between electron-electron and electron-phonon interactions have been restricted to the case where not only the electronic motion but also the phonon modes were strictly one dimensional [7]. For noninteracting electrons the phonon-induced resistance of quantum wires [8–11] and quantum point contacts [12] has also been investigated previously.

At temperatures low compared to the Fermi energy E_F a one-dimensional Fermi system can be viewed as consisting of two species of fermions, the right and left movers. By absorbing or emitting phonons the electrons can scatter either within each branch or between the branches. The intrabranch scattering leads to the establishment of thermal equilibrium within each branch but does not affect the resistivity of the wire. On the other hand, the interbranch scattering changes the electric current and thus gives rise to a finite resistivity. These processes involve phonons with the component of the wave vector along the wire $q_x \approx 2k_F$, where k_F is the Fermi wave vector. The minimal energy of such phonons is $T_A \equiv 2\hbar s k_F$, with s being the speed of sound. Thus at temperatures $T \ll T_A$ the phonon-induced resistivity is expected [11] to be exponentially small, $\rho = \rho_0 e^{-T_A/T}$. The prefactor ρ_0 is proportional to the square of the electron-phonon coupling constant, and for noninteracting electrons tends to be rather small [12]. We show below that repulsive interactions between electrons lead to a dramatic enhancement of ρ_0. In fact, since the typical energy transfer T_A in a given scattering event is negligible compared to the Fermi energy E_F, electron-phonon scattering is not dissimilar to scattering from static impurities. Borrowing the well-known result for that problem [6] one may conjecture that interactions enhance the result for noninteracting electrons by a factor of order $(E_F/T_A)^{1-K_\rho}$. Here the parameter K_ρ is determined by the interactions; repulsive interactions correspond to $K_\rho < 1$.

We now turn to the calculation of the resistivity ρ of a quantum wire caused by the scattering of electrons by acoustic phonons. We evaluate the electric field E in the wire, assuming that a dc current I flows through it, and define $\rho = dE/dI$. In a section of the wire of length L the electric field accelerates the electrons and increases their total momentum at the rate $\dot{P}_E = eEnL$, where n is electron density. In a stationary state, $\dot{P}_E$ is compensated by the momentum transfer $\dot{P}_{\rm ph}$ from the phonon subsystem, $\dot{P}_E + \dot{P}_{\rm ph} = 0$. Thus the electric field can be evaluated as

$$E = -\frac{1}{enL}\dot{P}_{\rm ph}. \tag{1}$$

The expression for $\dot{P}_{\rm ph}$ will be found in the second order of perturbation theory in the electron-phonon coupling

$$H_{\text{e-ph}} = \int_{-\infty}^{\infty} n(x)\Phi(x)\,dx, \tag{2}$$

where $n(x)$ is the operator of electron density at point x,

and $\Phi(x)$ is the electric potential created by the phonon-induced deformation of the crystal. In terms of the phonon creation and annihilation operators $\Phi(x)$ can be written as

$$\Phi(x) = \sum_{\mathbf{q}\lambda} \sqrt{\frac{\hbar}{2\rho_M V \omega_{\mathbf{q}\lambda}}} M_\lambda(\mathbf{q}) e^{iq_x x} (b_{\mathbf{q}\lambda} + b^\dagger_{-\mathbf{q}\lambda}). \quad (3)$$

Here the operator $b_{\mathbf{q}\lambda}$ destroys a phonon with wave vector $\mathbf{q}$, polarization λ, and frequency $\omega_{\mathbf{q}\lambda}$. The matrix element $M_\lambda(\mathbf{q})$ describes the coupling strength, ρ_M is the mass density of the material, and V is the sample volume.

The time derivative of the operator P of the system's momentum can be found from the Heisenberg equation of motion. The momentum change rate due to phonon scattering is $\dot{P}_{\text{ph}} = -(i/\hbar)[P, H_{\text{e-ph}}]$. The evaluation of the right-hand side of this equation gives

$$\dot{P}_{\text{ph}} = -\int_0^L n(x) \partial_x \Phi(x)\, dx.$$

Averaging this operator over the state of the wire perturbed in the first order in the electron-phonon coupling and using Eq. (1), we find

$$E = -\frac{i}{e\hbar n} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi_I(x, t) \partial_x \langle \Phi(x, t) \Phi(0, 0) \rangle\, dx dt.$$

Here $\chi_I(x, t) = \langle n(x, t) n(0, 0) \rangle_I$ is the electronic density-density correlator. The index I indicates that the expectation value is calculated with respect to a current-carrying state.

Using Eq. (3) one can easily evaluate the correlator $\langle \Phi(x, t) \Phi(0, 0) \rangle$ and find the following expression for the electric field in the wire:

$$\begin{aligned} E = \frac{1}{en} \sum_{\mathbf{q}\lambda} \frac{|M_\lambda(\mathbf{q})|^2}{2\rho_M V \omega_{\mathbf{q}\lambda}} q_x \\ \times \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dx \chi_I(x, t) e^{iq_x x} \{ N(\omega_{\mathbf{q}\lambda}) e^{i\omega_{\mathbf{q}\lambda} t} \\ + [N(\omega_{\mathbf{q}\lambda}) + 1] e^{-i\omega_{\mathbf{q}\lambda} t} \}. \end{aligned} \quad (4)$$

Here $N(\omega_{\mathbf{q}\lambda})$ is the equilibrium Bose occupation number of the phonon state $\{\mathbf{q}, \lambda\}$.

The typical phonon frequencies in Eq. (4) are of order T and small compared to the Fermi energy. At small frequencies, the density-density correlator can be presented as a sum of terms $e^{i2nk_F x} \chi^{(n)}$, where $\chi^{(n)}$ are slowly varying functions of x. Contribution of each of these terms to the electric field (4) accounts for the electron-phonon scattering processes that transfer $|n|$ electrons between the left- and right-moving branches. The term with $n = 0$ corresponds to intrabranch scattering, and to lowest order in T/E_F may be neglected. In the case of scattering by static impurities the processes with $n \neq 0$ have been studied in Refs. [6,13,14]. The repulsive interactions lead to the enhancement of the processes with $n = \pm 1$. Depending on the strength of the interactions, the remaining processes may be either suppressed or enhanced. For not too strong interactions the $|n| > 1$ terms can usually be neglected [6], and, for the moment, we therefore include only the $n = \pm 1$ components of the correlator χ in Eq. (4).

The functions $\chi^{(\pm 1)}(x, t)$ can be calculated in the framework of Luttinger liquid theory [1]. For electrons with spin and at finite temperature they are [15]

$$\begin{aligned} \chi^{(\pm 1)}(x, t) = \frac{1}{2\pi^2 \alpha^2} \left[\frac{(\alpha \pi T/\hbar u_\rho)^2}{\sinh \frac{\pi T y_\rho^+}{\hbar u_\rho} \sinh \frac{\pi T y_\rho^-}{\hbar u_\rho}} \right]^{K_\rho/2} \\ \times \left[\frac{(\alpha \pi T/\hbar u_\sigma)^2}{\sinh \frac{\pi T y_\sigma^+}{\hbar u_\sigma} \sinh \frac{\pi T y_\sigma^-}{\hbar u_\sigma}} \right]^{K_\sigma/2}, \end{aligned} \quad (5)$$

where $y_\nu^\pm = x \pm (u_\nu t - i\alpha)$, u_ρ and u_σ are the velocities of charge and spin excitations, and $\alpha \sim 1/k_F$ is a short distance cutoff. The dimensionless interaction parameters K_ρ and K_σ quantify the strength of the interactions in the charge and spin sector, respectively. In the following we are interested in the spin-rotation invariant case where $K_\sigma = 1$.

Note that Eq. (5) is valid in the rest frame of the electrons. In the presence of a driving current I, this frame moves with a constant drift velocity $v_d = I/ne$ relative to the laboratory frame. We therefore perform a Galilean transformation to the laboratory frame and replace $\chi_I(x, t)$ with $\chi(x - v_d t, t)$ [16]. We consider the case of low current, such that $v_d \ll s \ll u_\sigma, u_\rho$. Then one can neglect x in the argument of the slowly varying functions $\chi^{(\pm 1)}$ and substitute $\chi(x, t) = 2\cos[2k_F(x - v_d t)]\chi^{(1)}(0, t)$ into Eq. (4). In the linear response regime $I \to 0$ the electric field then becomes $E_1 = \rho_1 I$ where the resistivity is given by

$$\begin{aligned} \rho_1 = \frac{2\pi R_0}{\hbar u_\rho u_\sigma} \left(\frac{2\pi\alpha T}{\hbar u_\rho} \right)^{K_\rho - 1} \sum_\lambda \int \frac{d^3 q}{(2\pi)^3} \frac{|M_\lambda(\mathbf{q})|^2}{2\rho_M \omega_{\mathbf{q}\lambda}} \\ \times \frac{F_1(\frac{\hbar\omega_{\mathbf{q}\lambda}}{\pi T})}{\sinh \frac{\hbar\omega_{\mathbf{q}\lambda}}{2T}} \delta(q_x - 2k_F). \end{aligned} \quad (6)$$

Here we have introduced an auxiliary function

$$F_1(z) = \frac{|\Gamma(\frac{K_\rho + 1 + iz}{2})|^2}{\Gamma(K_\rho + 1)}$$

and the quantum resistance $R_0 = \pi\hbar/e^2$.

At low temperature the integrand in Eq. (6) is exponentially small as $e^{-\hbar\omega_{\mathbf{q}\lambda}/T}$, and thus the leading contribution to the resistivity is due to the phonons with the wave vector $\mathbf{q}_A = (2k_F, q_y^A, q_z^A)$ and polarization λ chosen in a way that minimizes the phonon energy $\hbar\omega_{\mathbf{q}\lambda}$. Denoting the energy of this phonon $\hbar\omega_A \equiv T_A$, the resistivity at $T \ll T_A$ is

$$\begin{aligned} \rho_1(T) = \frac{R_0}{\Gamma(K_\rho + 1)} \frac{\Upsilon(T)}{u_\rho u_\sigma} \left(\frac{\alpha T_A}{\hbar u_\rho} \right)^{K_\rho - 1} e^{-T_A/T}, \\ \Upsilon(T) = \sum_\lambda \int \frac{dq_y dq_z}{(2\pi)^2} \frac{|M_\lambda(\mathbf{q})|^2}{\rho_M T} e^{-(\hbar\omega_{\mathbf{q}\lambda} - T_A)/T} \Big|_{q_x = 2k_F}. \end{aligned} \quad (7)$$

From here on we concentrate on the experimentally most relevant case of a quantum wire embedded in a GaAs heterostructure. The electron-phonon coupling then has the form $M_\lambda(\mathbf{q}) = \delta_{\lambda,l} D|\mathbf{q}| + i\hat{M}_\lambda(\mathbf{q})$. Here D is the deformation potential for the longitudinal phonons, and $\hat{M}_\lambda(\mathbf{q})$ is the piezoelectric coupling which is strongly direction dependent [17].

At low temperatures $T \ll T_A$ the longitudinal phonon modes can be ignored. Indeed, the transverse sound velocity is smaller than the longitudinal one, $s \equiv s_t < s_l$, leading to a smaller value of the activation temperature $T_A = 2\hbar k_F s$. Furthermore, in this regime we approximate the piezoelectric couplings by their asymptotic values $|M_\lambda^<|^2 = |\hat{M}_\lambda(\mathbf{q_A})|^2$. With these approximations we obtain $\Upsilon(T) \equiv \Upsilon$, where

$$\Upsilon = \frac{n|M_t^<|^2}{\hbar\rho_M s}.$$

For noninteracting electrons the parameter $K_\rho = 1$, and velocities u_ρ, u_σ coincide with the Fermi velocity $v_F = \pi\hbar n/2m$. Thus the resistivity Eq. (7) takes the simple form $\rho_1 = \rho_0 e^{-T_A/T}$ with $\rho_0 = R_0\Upsilon/v_F^2$. For a numerical estimate we assume that the wire is oriented in the $[01\bar{1}]$ direction [3,18], where $|M_t^<|^2 = \frac{1}{4}(ee_{14}/\epsilon)^2$. In GaAs $s = 3\times 10^3$ m/s, the permittivity $\epsilon = 13.2\epsilon_0$, the mass density $\rho_M = 5.36$ g/cm^3, the effective electron mass $m = 0.067m_e$, and $e_{14} = 0.16$ C/m^2. For an electron density $n = 25\ \mu$m^{-1} the Fermi velocity is $v_F = 7\times 10^4$ m/s, and we estimate $\rho_0 = 0.03R_0\ \mu$m^{-1}. The activation temperature in this example is $T_A = 2$ K.

In an interacting wire one can estimate the velocities of the charge and spin excitations as $u_\rho = v_F/K_\rho$ and $u_\sigma = v_F$, respectively. With $\alpha \sim 1/k_F$ we see that interactions enhance the preexponential factor in Eq. (7) by $\sim(v_F/2s)^{1-K_\rho}$ relative to the noninteracting case. To estimate the interaction parameter we use [19]

$$K_\rho = \frac{1}{\sqrt{1 + (\frac{2}{\pi\hbar})^2 \frac{me^2}{nC}}},$$

where C is the capacitance per unit length of the wire relative to a nearby metallic gate. If the gate is modeled as an infinite conducting plane at a distance d from the wire, the capacitance is $C = 2\pi\epsilon/\ln(8.0nd)$ [20]. For a gate-wire distance $d = 100$ nm and electron density $n = 25\ \mu$m^{-1} the interaction parameter becomes $K_\rho = 0.3$. With this estimate the resistance enhancement is roughly $(v_F/2s)^{1-K_\rho} \sim 5$.

We now turn to the high temperature limit $T \gg T_A$ and first consider the case of low electron density n where interaction effects are expected to be most pronounced. Assuming $T \ll \hbar u_\rho/\alpha \sim E_F/K_\rho$, from Eq. (6) we obtain

$$\rho_1(T) = R_0 \frac{\tilde{\Upsilon} F_1(0)}{u_\rho u_\sigma}\left(\frac{2\pi\alpha T}{\hbar u_\rho}\right)^{K_\rho - 1} \frac{T}{T_A}\ln\left(\frac{T}{T_A}\right). \quad (8)$$

Again, interactions are found to enhance the phonon-induced resistivity. Furthermore, in this regime a measurement of the temperature dependence of the resistivity provides direct information about the interaction parameter K_ρ. The parameter $\tilde{\Upsilon}$ in Eq. (8) is defined as

$$\tilde{\Upsilon} = \sum_\lambda \frac{n|M_\lambda^>|^2 s}{2\hbar\rho_M s_\lambda^2},$$

where both the transverse and longitudinal phonon modes are now included. The piezoelectric coupling functions were approximated by their high energy asymptotes $|M_\lambda^>|^2 = \lim_{q_\perp/k_F\to\infty}\langle|\hat{M}_\lambda(2k_F, q_\perp\cos(\phi), q_\perp\sin(\phi))|^2\rangle_\phi$, where the angular brackets indicate angle averaging over directions perpendicular to the wire. Such an approximation is possible since scattering in the regime $T \gg T_A$ is dominated by phonons with $q \gg 2k_F$ and $q_x = 2k_F$. For the $[01\bar{1}]$ direction we find $|M_t^>|^2 = \frac{5}{64}(ee_{14}/\epsilon)^2$ and $|M_l^>|^2 = \frac{9}{16}(ee_{14}/\epsilon)^2$. With $s_l = 5.2\times 10^3$ m/s for the longitudinal sound velocity in the $[01\bar{1}]$ direction, we find $R_0\tilde{\Upsilon}/v_F^2 = 0.03R_0\ \mu$m^{-1}. When deriving Eq. (8) we neglected coupling via the deformation potential assuming that $q \ll |ee_{14}/(\epsilon D)|$. For low enough electron densities this is consistent with $q \gg 2k_F = q_x$ [21].

Up to here, we treated electrons as purely one dimensional; i.e., the width of the wire w was assumed to be small compared to k_F^{-1}. This condition is not satisfied on the high-density side of the first conductance plateau, where $k_F \sim 1/w$. If we allow for a lateral extension of the electron wave functions of order w, an effective momentum cutoff $\sim 1/w$ is introduced into Eq. (6). The resistivity can then be calculated from these modified equations, and in the limit $T \gg T_A$ we find

$$\rho_1(T) = R_0 \frac{\Upsilon_D F_1(0)}{A u_\rho u_\sigma}\left(\frac{2\pi\alpha T}{\hbar u_\rho}\right)^{K_\rho - 1} \frac{T}{T_A}. \quad (9)$$

The parameter Υ_D and the effective wire cross section A are defined as

$$\Upsilon_D = \frac{\pi D^2 ns}{\hbar\rho_M s_l^2}, \qquad \frac{1}{A} = \int dydz|\psi(y,z)|^4,$$

where $\psi(y,z)$ is the wave function for transverse motion in the lowest subband. In the high-density regime discussed here it is safe to assume that coupling is mainly via the deformation potential. For noninteracting electrons the resistivity takes the simple form $\rho_1 = \rho_0 T/T_A$, where $\rho_0 = R_0\Upsilon_D/(Av_F^2)$. At a density $n = 100\ \mu$m^{-1} and $A = 100$ nm^2 we get $\rho_0 = 0.01R_0\ \mu$m^{-1}. Here we used $D = 7$ eV for the deformation potential in GaAs.

So far we have limited the discussion to $2k_F$ scattering where only a single electron is transferred between the left- and right-moving branches in any given scattering event. This is justified if interactions are not too strong. To understand this we consider the long-time limiting behavior of the density-density correlator. From Eq. (5) we see that $\chi^{(\pm 1)} \propto 1/t^{K_\rho + K_\sigma}$ while the next contribution is known [1]

to decay as $\chi^{(\pm 2)} \propto 1/t^{4K_\rho}$. This second term thus dominates at $K_\rho < 1/3$. This is due to the absence of K_σ from the exponent of the $n = \pm 2$ terms, which can be understood from the fact that in a two-particle scattering process it is possible to transfer a pair of electrons with zero total spin. Contributions to the density-density correlator with $|n| > 2$ always decay faster than the two leading contributions and can therefore be neglected.

For purely elastic scattering, as is the case for static impurities [1], the $4k_F$ contribution would thus dominate at $K_\rho < 1/3$. In the case of electron-phonon scattering in the low-temperature regime $T \ll T_A$, however, the resistivity due to $4k_F$ scattering is negligibly small. Indeed, the activation energy for a process with a momentum transfer $4k_F$ is $2T_A$, and such processes are thus exponentially suppressed relative to $2k_F$ scattering.

On the other hand, at temperatures $T \gg T_A$ the $4k_F$ scattering cannot be neglected. The resistivity then is the sum of a contribution $\rho_1(T)$ due to scattering events with a $2k_F$-momentum transfer, and a contribution $\rho_2(T)$ due to $4k_F$ scattering. To calculate the latter we proceed in exactly the same way that led us to Eq. (8) but this time use $\chi^{(\pm 2)}(x, t)$ instead of $\chi^{(\pm 1)}(x, t)$. Up to a constant which cannot be determined within the Luttinger liquid approach, the functions $\chi^{(\pm 2)}(x, t)$ are obtained from Eq. (5) by substituting $K_\sigma \to 0$ and $K_\rho \to 4K_\rho$ [1]. As a result we find $\rho_2(T) \sim \rho_1(T)(D_\rho/T)^{3K_\rho - 1}$, where $D_\rho = \hbar u_\rho/\alpha$ is the bandwidth of charge excitations. As expected, scattering processes with a $2k_F$-momentum transfer dominate for weak interactions, while $4k_F$ scattering is most relevant if interactions are strong, $K_\rho < 1/3$.

We have shown that electron-electron interactions strongly enhance the phonon-induced resistivity of a quantum wire. A measurement of the temperature dependence of the resistivity provides information about the strength of these interactions. The phonon-induced resistivity can be explored experimentally in a four-terminal measurement similar to that of Ref. [22]. Since phonon effects can be subtle, in a two-terminal measurement they are best observed for reflectionless coupling between the one-dimensional wire and the two-dimensional leads when the contact resistance takes the constant value $R_0 = \pi\hbar/e^2$ independent of temperature [23]. Prior experimental work [2,3] does not allow us to unambiguously extract phonon effects. The temperature dependence of the resistance reported in Ref. [2] is most probably due to nonideal coupling between the two-dimensional leads and the wire while the experiment of Ref. [2] was done at very low temperatures where phonon effects are expected to be exponentially small [cf. Eq. (7)].

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